Intermediate Microeconomics-I -Sem 3

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Intertemporal Choice

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Objective

- Consumer lives in two period : period 1 and period 2
  C_1 : consumption at period 1
  C_2 : consumption at period 2
  $U(C_1)$: Utility from consumption C_1 i.e, at period 1
  $U(C_2)$: Utility from consumption C_2 i.e, at period 2
- If both period are equally important then total lifetime utility (V) must be equal to $U(C_1) + U(C_2)$.
- But most of the consumer prefer present consumption than the future so gives more value to present than future. To put this idea in a math let β is the time preference parameter of period 2, $\beta < 1$ implies they put more value for present (which is one) than the future. That is they are impatient.

- Hence total utility (lifetime utility) $V = U(C_1) + \beta U(C_2)$
- Note that $U(.)$ function is called felicity function and V is called lifetime utility function.
- Lifetime utility function is a additive separable. Which means marginal rate of substitution(MRS) between any two period (say t and t+1) is independent of any other period.
- **Assignment 1 :** Suppose $V = U(C_1) + \beta U(C_2) = C_1^{\alpha} + \beta C_1^{\alpha} C_2^{(1-\alpha)}$ Is this additive separable ?

- Concavity : $U'(C_t) > 0$ and $U''(C_t) < 0$ MU_{c_t} falls as C_t goes up.
- Example $U(C_1) = \frac{C_1^{(1-\sigma)}}{1-\sigma}$ and $U(C_2) = \frac{C_2^{(1-\sigma)}}{1-\sigma}$ where $\sigma > 0$. This is type of CRRA utility function.
- **Assignment 2 :** Check whether felicity function $U(C_1)$ or $U(C_2)$ is concave.

Budget line

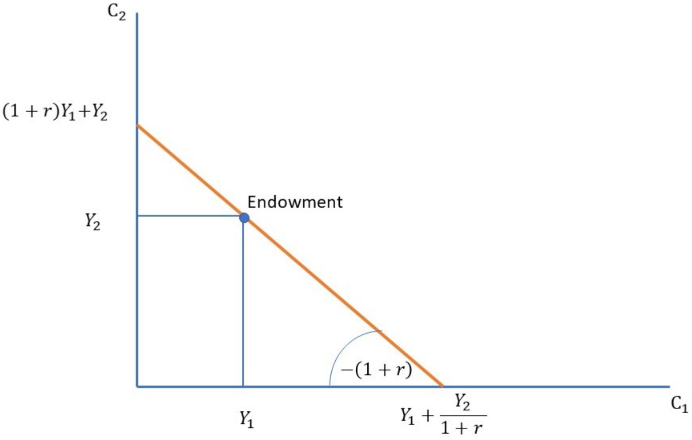
- Y_1 - Income in Period 1 , Y_2 - Income in period 2 Therefore $S = Y_1 - C_1$ If r is the rate of return (interest rate) then in period 2 he will earn rS as interest income and since period 2 is the last period so his principal plus interest income must be equal to $(1+r)S$
- Hence maximum amount of consumption for period 2 is

$$\begin{aligned} C_2 &= Y_2 + (1+r)S \\ \implies C_2 &= Y_2 + (1+r)(Y_1 - C_1) \\ \implies C_2 + (1+r)C_1 &= Y_2 + (1+r)Y_1 \quad (1) \\ \implies C_1 + \frac{C_2}{(1+r)} &= Y_1 + \frac{Y_2}{(1+r)} \quad (2) \end{aligned}$$

This is lifetime budget constraint. We say that equation (1) expresses the budget constraint in terms of **future value** and that equation (2) expresses the budget constraint in terms of **present value**.

Budget line : Graph

- The budget lien passes through (Y_1, Y_2) since this is always an *affordable* consumption. And the budget line has a slope $\frac{dC_2}{dC_1} = -(1+r)$ (from equation (1))



Optimization

- So our problem is

$$\begin{aligned} &\text{Max}_{\{C_1,C_2\}}\ U(C_1)+\beta U(C_2) \\ &\text{Subject to}\quad C_1+\frac{C_2}{(1+r)} = Y_1+\frac{Y_2}{(1+r)} \end{aligned} \tag{3}$$

The Lagrangian is

$$\mathcal{L}=U(C_1)+\beta U(C_2)+\lambda\left[Y_1+\frac{Y_2}{1+r}-C_1-\frac{C_2}{1+r}\right]$$

- Differentiating with respect to C_1 , C_2 & λ we'll have

$$\frac{\partial\mathcal{L}}{\partial C_1}=0: U'(C_1)-\lambda = 0 \tag{4}$$

$$\frac{\partial\mathcal{L}}{\partial C_2}=0: \beta U'(C_2)-\frac{\lambda}{1+r} = 0 \tag{5}$$

$$\frac{\partial\mathcal{L}}{\partial\lambda}=0: Y_1+\frac{Y_2}{1+r}-C_1-\frac{C_2}{1+r} = 0$$

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Interpretation

- Dividing 4 by 5 we have

$$\begin{aligned} \frac{U'(C_1)}{U'(C_2)} &= \beta(1+r) \qquad (6) \\ \implies MU_1 &= \beta(1+r)MU_2 \end{aligned}$$

This is famous Euler's equation , opportunity cost of C_1 in terms of C_2. If you consume one more unit of C_1 at present you will loose $(1+r)$ unit of future consumption.

- As r increases present consumption becomes more expensive. One will try to move towards future consumption.
- Similarly as β increases you are more patient and will move towards future consumption.

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Explicit Solution

- For any specific utility function we can explicitly solve for 6. Let us take the following example The lifetime utility function is

$$V = \frac{C_1^{(1-\sigma)}}{1-\sigma} + \beta \frac{C_2^{(1-\sigma)}}{1-\sigma}$$

$$\text{Then} \quad \frac{U'(C_1)}{U'(C_2)} = \beta(1+r) \text{ implies}$$

$$\begin{aligned} \frac{C_1^{-\sigma}}{C_2^{-\sigma}} &= \beta(1+r) \\ \Longrightarrow C_2^{\sigma} &= \beta(1+r)C_1^{\sigma} \end{aligned}$$

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Comparative Statics

- From this and budget line you can explicitly solve for C_1 and C_2 as

$$C_1^* = \frac{1}{1+\beta^{1/\sigma}(1+r)^{1/\sigma-1}}\left(Y_1+\frac{Y_2}{1+r}\right)$$
$$C_2^* = \frac{\beta^{1/\sigma}(1+r)^{1/\sigma}}{1+\beta^{1/\sigma}(1+r)^{1/\sigma-1}}\left(Y_1+\frac{Y_2}{1+r}\right)$$

- Differentiating above equations with respect to β , r , Y_1 and Y_2, we can calculate $\frac{\partial C_i^*}{\partial \beta}$ $\frac{\partial C_i^*}{\partial r}$, $\frac{\partial C_i^*}{\partial Y_j}$ $\forall$i &j $\in \{1,2\}$
- **Assignment 3** : Check the sign of these derivatives.

Graphical Analysis

- Given a consumer's budget constraint and his preferences for consumption in each of the two periods, we can examine the optimal choice of consumption (C_1, C_2).
- If consumer chooses a point where $C_1 < Y_1$ we will say that she is a **lender**, and if $C_1 > Y_1$ we say that she is **borrower**.

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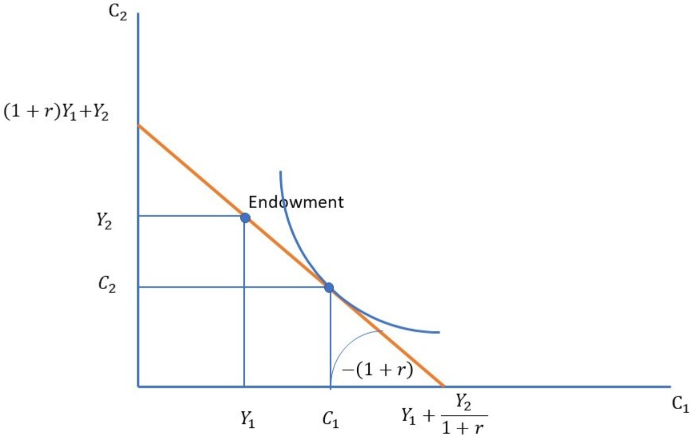
Graphical Analysis : Borrower



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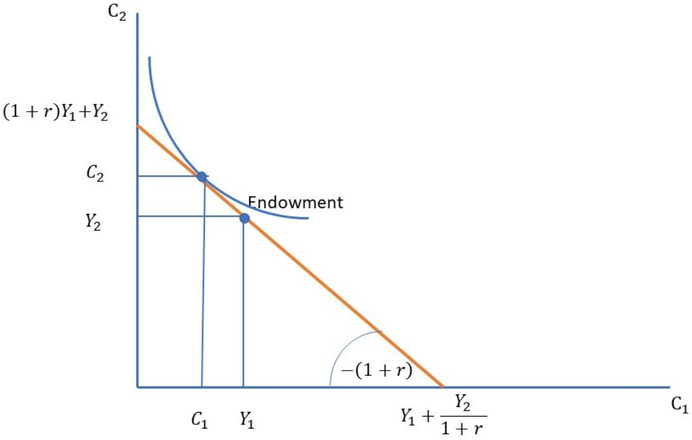
Graphical Analysis : Lender



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Comparative Statics : effect of increase in Interest rate on Borrowing

- When rate of interest increases from r_0 to r_1 the original (orange budget) line revolves pivoting endowment point to red line.
- If new equilibrium is established at G, he must be worse off (movement from higher IC(U_2) to lower IC (U_1)), because this point was affordable under old budget set (area below the orange line) but was rejected.
- SE : C_1B, IE: BD and TE :C_1D , note that SE and IE work in same direction. When the interest rate rise, there is always a SE towards consuming less today. For a borrower, an increase in the interest rate means that he will have to pay more interest tomorrow. This effect induces him to borrow less and thus consume less in the first period

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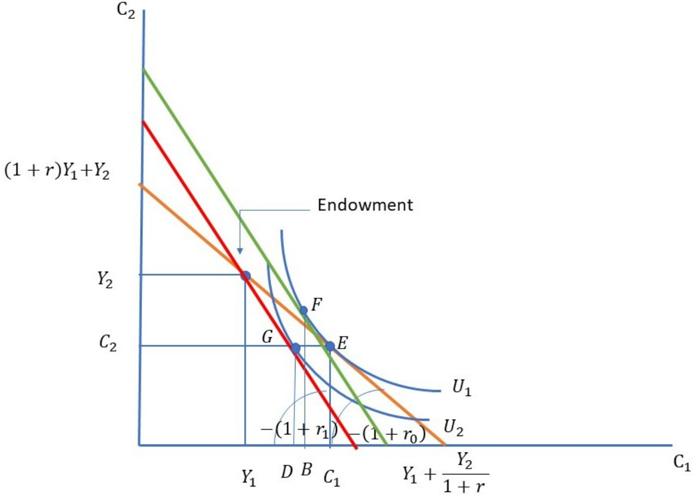
Comparative Statics : effect of increase in Interest rate on Borrowing



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Comparative Statics : effect of increase in Interest rate on Lending

- On the other hand, if the person is lender and interest rate has increased from r_0 to r_1 the original (orange budget) line revolves pivoting endowment point to red line.
- If new equilibrium is established at G, he must be better off (movement from lower IC,(U_1) to higher IC (U_2)).
- SE : C_1B, IE: BD and TE :C_1D , note that SE and IE work in opposite direction . When the interest rate rise, there is always a SE towards consuming less today. For a lender, an increase in the interest rate means that he will earn more interest interest income tomorrow. This effect induces him to consume more in the first period.

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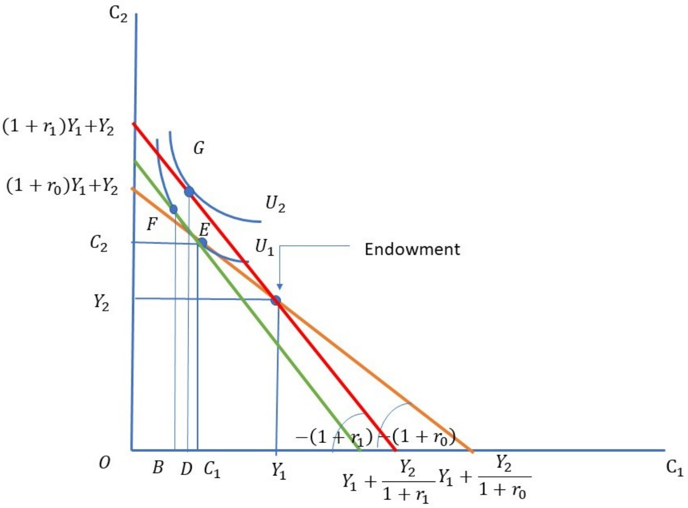

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Comparative Statics : effect of increase in Interest rate on Lending

- Although the net effect is ambiguous (depends on the relative strength of Negative Substitution effect and positive income effect), but It is for sure that he will never go (right) beyond the endowment. Because going beyond means violating the principle of revealed preference.
- **Assignment 4** : Find the effect of decrease in interest rate on Lending and borrowing.

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Comparative Statics : Two different interest rate

- **Assignment 5** : Draw the budget lines of a consumer for consumption in year 1 and consumption in year 2 in the following cases :
  (a) Income in year 1 is Rs. 1000, expected income in year 2 is Rs. 1100 and there are no borrowing and lending opportunities.
  (b)Incomes are the same as in (a) but there exists credit market where the annual interest rate is 10 percent; and
  (c) incomes are the same as in (a) but there are two interest rates in the credit market - 10% for lending and 15% for borrowing.

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Assignments

- **Assignment 6** :A consumer survives for just two time periods 1 and 2. The consumer gets income M_1 and M_2 in the two periods and consumes C_1 and C_2. The consumer can reallocate consumption between the two periods by saving and borrowing at the market rate of interest i. If both C_1 and C_2 are normal goods and the second period's income (M_2) falls, then in which direction will the budget line shift?— Why? (5 marks, CU-2020)

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Assignments

- **Assignment 7** : A consumer has the utility function: $u(c_1, c_2) = c_1c_2$, where c_1 and c_2 are the consumption in period 1 and 2, respectively. He earns an income of Rs. 100,000 in period 1 and Rs. 129,600 in period 2. If the objective is to optimize the consumption choice over time, work out the required consumption in each period and determine whether he would need to borrow or lend ?

  a) Assume that the rate of interest is 8% per annum and there is no inflation. (6 marks, DU 2017).

  b) Assume that the rate of interest is 8% per annum and the rate of inflation is 3% per annum. (Hints : $\frac{U_1}{U_2} = (1+\rho)$ where $\rho \approx (r - \pi)$, $\pi =$ rate of inflation, and $r =$ nominal rate of interest. Or more accurately $\rho = \frac{r-\pi}{1+\pi}$)

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- (2) Hal R Varian : Intermediate Microeconomics , Chapter 10
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Revealed preference

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Concept of Revealed Preference

- Up until now we were thinking about what preferences could tell us about people's behaviour. But in real life, preferences are not directly observable.
- We have to discover people's preferences from observing their behaviour. In this section we will develop some tools to do this.
- Let $X^0(x_1^0, x_2^0)$ be the bundle purchased at price $P^0(p_1^0, p_2^0)$ when the consumer has income m. Another bundle $X^1(x_1^1, x_2^1)$ is affordable at that price $P^0(p_1^0, p_2^0)$. Then it must be

$$P^0 X^0 \geq P^0 X^1$$

i.e.,

$$p_1^0 x_1^0 + p_2^0 x_2^0 \geq p_1^0 x_1^1 + p_2^0 x_2^1$$

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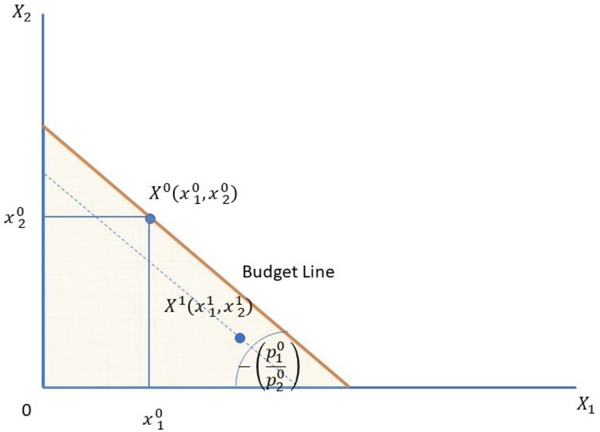
Concept of Revealed Preference



- If above inequality is satisfied and $X^1(x_1^1, x_2^1)$ is different from $X^0(x_1^0, x_2^0)$, we say that $X^0(x_1^0, x_2^0)$ is **directly revealed preferred to** $X^1(x_1^1, x_2^1)$, i.e., $X^0 R^D X^1$

From revealed preference to preference

- **The Principle of Revealed Preferences** . Let (x_1^0, x_2^0) be the chosen bundle when prices are (p_1^0, p_2^0), and let (x_1^1, x_2^1) be some other bundle such that $p_1^0 x_1^0 + p_2^0 x_2^0 \geq p_1^0 x_1^1 + p_2^0 x_2^1$. Then if the consumer is choosing the most preferred bundle she can afford, we must have $(x_1^0, x_2^0) \succ (x_1^1, x_2^1)$.
- Now suppose that we happen to know that (x_1^1, x_2^1) is demanded bundle at prices (p_1^1, p_2^1) and that (x_1^1, x_2^1) is revealed preferred to some other bundle (x_1^2, x_2^2). That is

$$p_1^1 x_1^1 + p_2^1 x_2^1 \geq p_1^1 x_1^2 + p_2^1 x_2^2$$

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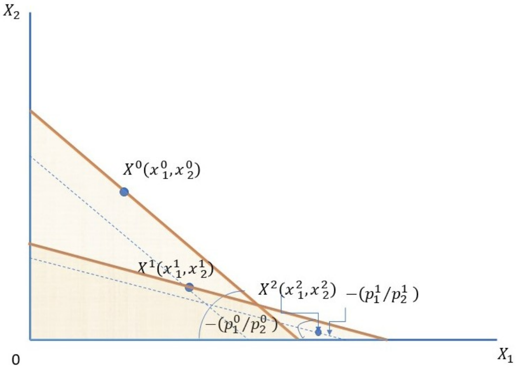
From revealed preference to preference

- then we know that $(x_1^0, x_2^0) \succ (x_1^1, x_2^1)$ and that $(x_1^1, x_2^1) \succ (x_1^2, x_2^2)$. From the transitivity assumption we can conclude that $(x_1^0, x_2^0) \succ (x_1^2, x_2^2)$. It is natural to say that in this case (x_1^0, x_2^0) is **indirectly revealed preferred** to (x_1^2, x_2^2).



From revealed preference to preference

- If a bundle is either directly or indirectly revealed preferred to another bundle, we will say that the first bundle is **revealed preferred** to the second.
- The idea of revealed preference is simple, but it is surprisingly powerful. For example consider the figure above. From the figure we can conclude that since (x_1^0, x_2^0) is revealed preferred, either directly or indirectly, to all of the bundle in the shaded area. And hence IC through (x_1^0, x_2^0) must lie above the shaded region.

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Weak Axioms of Revealed Preference (WARP)

- **Weak Axioms of Revealed Preference (WARP).** *If (x_1^0, x_2^0) is directly revealed preferred to (x_1^1, x_2^1), and the two bundles are not the same, then it can't happen that (x_1^1, x_2^1) is directly revealed preferred to (x_1^0, x_2^0).*
- In other word if a bundle (x_1^0, x_2^0) is purchased at prices (p_1^0, p_2^0) and a different bundle is purchased at prices (p_1^1, p_2^1) , then if

$$p_1^0 x_1^0 + p_2^0 x_2^0 \geq p_1^0 x_1^1 + p_2^0 x_2^1$$

It must **not** be the case that

$$p_1^1 x_1^1 + p_2^1 x_2^1 \geq p_1^1 x_1^0 + p_2^1 x_2^0$$

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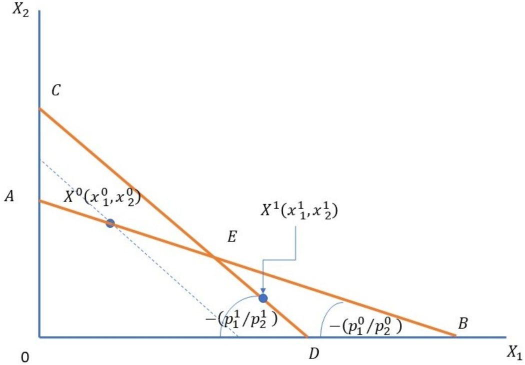
Weak Axioms of Revealed Preference (WARP)



WARP is Violated

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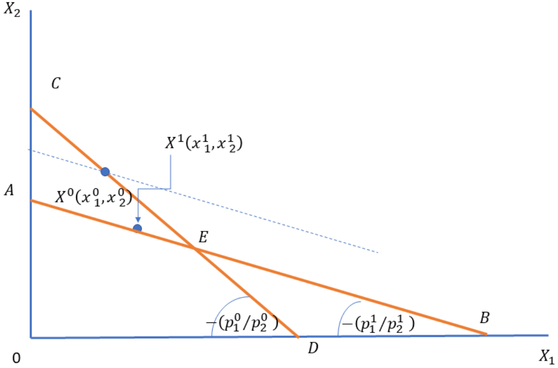
Weak Axioms of Revealed Preference (WARP)



WARP is Satisfied

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Checking Weak Axioms of Revealed Preference (WARP) : Numerical Example

- Example 1 : When Prices are $(p_1^0, p_2^0) = (1, 2)$ a consumer demands $(x_1^0, x_2^0) = (1, 2)$ and when prices are $(p_1^1, p_2^1) = (2, 1)$ the consumer demands $(x_1^1, x_2^1) = (2, 1)$. Is this behaviour consistent with the model of maximizing behaviour ?
  Sol: Here

$$\begin{aligned}
p_1^0 x_1^0 + p_2^0 x_2^0 &= 1 \times 1 + 2 \times 2 = 5 \\
p_1^0 x_1^1 + p_2^0 x_2^1 &= 1 \times 2 + 2 \times 1 = 4 \\
\therefore p_1^0 x_1^0 + p_2^0 x_2^0 &> p_1^0 x_1^1 + p_2^0 x_2^1 \\
\text{Again} \quad p_1^1 x_1^1 + p_2^1 x_2^1 &= 2 \times 2 + 1 \times 1 = 5 \\
p_1^1 x_1^0 + p_2^1 x_2^0 &= 2 \times 1 + 1 \times 2 = 4 \\
\therefore p_1^1 x_1^1 + p_2^1 x_2^1 &> p_1^1 x_1^0 + p_2^1 x_2^0
\end{aligned}$$

WARP is violated

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Checking Weak Axioms of Revealed Preference (WARP) : Step by Step Procedure

- Let us follow this two step procedure:
  1.Checking the Premises : Check bundles X^0 and X^1 lie on or below the initial budget line AB, which represent initial prices (p_1^0, p_2^0). That is make sure that both bundles are affordable.
- 1.a If step 1 holds move to step 2.
- 1.b If step 1 does not hold, then stop. We can only claim that individual choices do not violate WARP.
- **2.Checking the Conclusion** : Check if bundle X^0 lies strictly above the final budget line CD which represents final prices (p_1^1, p_2^1). That is check that bundle X^0 is no longer affordable
- 2.a If step 2 holds, then WARP is *satisfied*
- 2.b If step 2 does not hold, then WARP is *violated*.

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Checking Weak Axioms of Revealed Preference (WARP) : Numerical Example

- **Assignment 1** : When Prices are $(p_1^0, p_2^0) = (2, 1)$ a consumer demands $(x_1^0, x_2^0) = (1, 2)$ and when prices are $(p_1^1, p_2^1) = (1, 2)$ the consumer demands $(x_1^1, x_2^1) = (2, 1)$. Is this behaviour consistent with the model of maximizing behaviour ?
- **Assignment 2** : 2 pens and 4 pencils are bought when prices of pens and pencils are ₹ 2 and ₹ 4 respectively. When price of pens rises to ₹ 4 and price of pencils fall to ₹ 2, the quantities of pens and pencils bought are 4 pens and 2 pencils. Do these observations indicate violation of Weak Axiom of Revealed Preference Theory? (5 marks, CU-2020)

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The Strong Axioms of Revealed Preference (SARP)

- The weak axiom of revealed Preferences requires that if X^0 is directly revealed preferred to X^1 then we should never observe X^1 being *directly* revealed preferred to X^0. The Strong Axiom of Revealed Preference(SARP) requires that the same sort of condition hold for *indirect* revealed preference.
- **Strong axiom of Revealed Preference (SARP).** If (x_1^0, x_2^0) is revealed preferred to (x_1^1, x_2^1) (either directly or indirectly) and (x_1^1, x_2^1) is different from (x_1^0, x_2^0) , then (x_1^1, x_2^1) can not be directly or indirectly revealed preferred to (x_1^0, x_2^0).

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The Strong Axioms of Revealed Preference (SARP)

- 1. WARP is only a necessary condition for behavior to be consistent with utility maximization
- 2. Strong Axiom of Revealed Preference (SARP): if (x_1^0, x_2^0) is directly or indirectly revealed preferred to (x_1^1, x_2^1), then (x_1^1, x_2^1) cannot be directly or indirectly revealed preferred to (x_1^0, x_2^0)
- 3. SARP is a necessary and sufficient condition for utility maximization
- 4. this means that if the consumer is maximizing utility, then his behavior must be consistent with SARP
- 5. furthermore if his observed behavior is consistent with SARP, then we can always find a utility function that explains the behavior of the consumer as maximizing behavior.

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Checking Strong Axioms of Revealed Preferences (SARP)

- Is the following set of observation of price-quantity data are consistent with utility maximization?

$$
\begin{aligned}
P^1 &= (1,2,3), \quad X^1 = (3,2,1) \\
P^2 &= (2,1,2), \quad X^2 = (2,2,1) \\
P^3 &= (3,5,1), \quad X^3 = (1,2,1)
\end{aligned}
$$

- Sol:

$$
\begin{aligned}
P^1X^1 &= 1\times 3 + 2\times 2 + 3\times 1 = 10 \\
P^1X^2 &= 2\times 2 + 1\times 2 + 2\times 2 = 10 \\
P^2X^2 &= 2\times 2 + 1\times 2 + 2\times 1 = 8 \\
P^2X^1 &= 2\times 3 + 1\times 2 + 2\times 1 = 10 \\
P^3X^3 &= 3\times 1 + 5\times 2 + 1\times 1 = 14
\end{aligned}
$$

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Checking Strong Axioms of Revealed Preferences (SARP)

$$\begin{aligned} P^3X^2 &= 3 \times 2 + 5 \times 2 + 1 \times 1 = 17 \\ P^2X^3 &= 2 \times 1 + 1 \times 2 + 2 \times 1 = 6 \end{aligned}$$

If $P^1X^1 \geq P^1X^2$ & $P^2X^2 < P^2X^1$ then $X^1R^DX^2$. Here all conditions are satisfied hence $X^1R^DX^2$. WARP is satisfied.If $P^2X^2 \geq P^2X^3$ & $P^3X^3 < P^3X^2$ then $X^2R^DX^3$.Here all conditions are satisfied hence $X^2R^DX^3$. WARP is satisfied.

$$\begin{aligned} P^1X^3 &= 1 \times 1 + 2 \times 2 + 3 \times 1 = 8 \\ P^3X^1 &= 3 \times 3 + 5 \times 2 + 1 \times 1 = 20 \end{aligned}$$

If $P^1X^1 \geq P^1X^3$ & $P^3X^3 < P^3X^1$ then X^1RX^3.Here all conditions are satisfied hence X^1RX^3. Hence SARP is satisfied.

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Assignments

- An individual consumes three goods x_1, x_2 and x_3 at respective prices p_1, p_2 and p_3. His month-wise consumption amounts of x_i at prices p_i in three different months are given in each rows of the table below :

	x_1	x_2	x_3	p_1	p_2	p_3
Month 1	3	2	4	2	3	6
Month 2	4	2	3	4	1	7
Month 3	3	7	2	3	2	1

Check if this price and consumption data is consistent with :
(a) weak axiom of revealed preference, and (b) strong axiom of revealed preference ?

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Assignments

- Sol: Hints

	x_1	x_2	x_3	p_1	p_2	p_3	Exp. at month 1	Exp. at month 2	Exp. at month 3
Month 1	3	2	4	2	3	6	$2 \times 3 + 3 \times 2 + 6 \times 4 = 36$	32	39
Month 2	4	2	3	4	1	7	$4 \times 3 + 1 \times 2 + 7 \times 4 = 42$	39	33
Month 3	3	7	2	3	2	1	$3 \times 3 + 2 \times 2 + 1 \times 4 = 17$	19	25

- WARP is satisfied and SARP is violated.

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Substitution Effect is negative: Proof

- It can be proved from Revealed Preference theory that the substitution effect is negative.
- **Proof :** Make the following assumptions
  (i) The consumer is indifferent between X^0 and X^1
  (ii) He chooses X^0 when prices are P^0 and his income is P^0X^0
  (iii) He chooses X^1 when prices are P^1 and his income is P^1X^1
  From (ii) we can infer that when prices are P^0, X^1 must be at least as expensive as X^0 , since if it were cheaper, he would have chosen it rather than X^0. So

$$P^0X^0 \leq P^0X^1 \implies \sum_i p_i^0 x_i^0 \leq \sum_i p_i^0 x_i^1$$

$$\therefore \textstyle\sum_i -p_i^0(x_i^1 - x_i^0) \leq 0$$

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Substitution Effect is negative: Proof

- From (iii) we can infer that when prices are P^1, X^0 must be at least as expensive as X^1 , since if it were cheaper, he would have chosen it rather than X^1. So

$$P^1X^1 \leq P^1X^0 \implies \sum_i p_i^1 x_i^1 \leq \sum_i p_i^1 x_i^0$$

$$\therefore \sum_i p_i^1(x_i^1 - x_i^0) \leq 0$$

Therefore adding our second and fourth equation we find that

$$\sum_i (p_i^1 - p_i^0)(x_i^1 - x_i^0) \leq 0$$

If only p_1 has changed it follows that

$$(p_1^1 - p_1^0)(x_1^1 - x_1^0) \leq 0$$

So if p_1 has risen, x_1 has either fallen or remained constant.This proves that SE is non-positive.

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Substitution Effect is negative: Proof

- By the assumption that the price changes are non zero and that x_1^1 and x_1^0 are distinct, the above inequality must be strict i.e,

$$(p_1^1 - p_1^0)(x_1^1 - x_1^0) < 0$$

- This proves that the substitution effect is negative.
- Hence WARP implies **downward-sloping (compensated) demand.** No need for assumptions on quasi-concavity of Utility function or MRS.

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Application: Goods Tax (Indirect Tax) vs Income tax (Direct Tax)

- Either good 1 is taxed at the rate t (non uniform sales tax) such that its new price is $= p_1 + t$
- Or income is taxed with lumpsum amount L, such that amount of collection of the taxes are same , i.e, $L = tx_1^T$ (where x_1^T denotes the new level of consumption with tax t).
- **Question is : which tax is better ?**

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Application: Goods Tax (Indirect Tax) vs Income tax (Direct Tax)

- Let's (x_1, x_2) denotes the consumption bundle before tax, (x_1^L, x_2^L) with the lump-sum tax, and (x_1^T, x_2^T) with the sales tax.
- Initial budget constraint implies: $p_1x_1 + p_2x_2 = m$
- Lump-sum tax implies: $p_1x_1^L + p_2x_2^L = m - L$
- Sales tax implies: $(p_1 + t)x_1^T + p_2x_2^T = m$
- Combining these three equations together with $L = tx_1^T$ we obtain that x_1^T, x_2^T is also on the budget line after the lump-sum tax with undistorted prices (p_1, p_2): $p_1x_1^L + p_2x_2^L = p_1x_1^T + p_2x_2^T$

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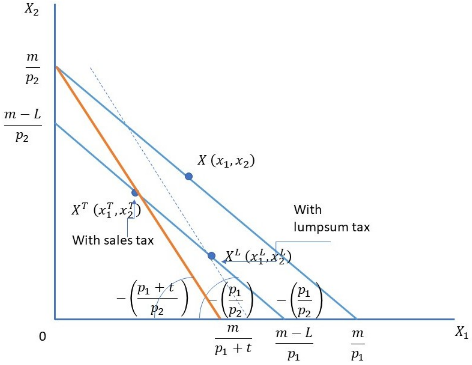
Application: Goods Tax (Indirect Tax) vs Income tax (Direct Tax)

- WARP implies that consumers are better off with lump-sum tax.



Consequences of Taxation

Application: Goods Tax (Indirect Tax) vs Income tax (Direct Tax)

- Why is the lump-sum tax L better than sales tax t ?
- Ans. Sales tax distorts optimal consumption baskets:

$$\frac{U_1}{U_2} = \frac{p_1 + t}{p_2} \neq \frac{p_1}{p_2}$$

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Index Number

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Concept of Index Number

- Index number is a measure to examine how things change overtime.
- So we have a **base period** (0) with which we want to compare **current period** (1).
- For example, if we want to see the price movement, how price has changed overtime, we calculate **price index number**. if we want to see the quantity movement overtime we calculate **quantity index number**.
- If we want to calculate cost of living then we can calculate the cost of living index number etc.

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Classification of Index Number

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Price Index

- Index number can be classified in two major groups. (1) Aggregative , (2) Relative. Each of them is again classified into two groups (i) Unweighted and (ii) Weighted.
- **(1) Aggregative - (i) Unweighted :**

$$I_{01}^{uw} = \frac{p_1^1 + p_1^2 + \ldots\ldots}{p_0^1 + p_0^2 + \ldots\ldots} \times 100 = \frac{\sum_i p_1^i}{\sum_i p_0^i} \times 100$$

- **- (ii) Weighted :**

$$I_{01}^{w} = \frac{p_1^1 w_1 + p_1^2 w_2 + \ldots\ldots}{p_0^1 w_1 + p_0^2 w_2 + \ldots\ldots} \times 100 = \frac{\sum_i p_1^i w_i}{\sum_i p_0^i w_i} \times 100$$

Classification of Index Number

Price Index

- **Laspeyres Price Index (L_{01}^{P}) : weights are base year quantity** (q_0)

$$L_{01}^{P} = \frac{p_1^1 q_0^1 + p_1^2 q_0^2 + \ldots\ldots}{p_0^1 q_0^1 + p_0^2 q_0^2 + \ldots\ldots} \times 100 = \frac{\sum_i p_1^i q_0^i}{\sum_i p_0^i q_0^i} \times 100$$

- **Paasche Price Index (P_{01}^{P}) : weights are current year quantity** (q_1)

$$P_{01}^{P} = \frac{p_1^1 q_1^1 + p_1^2 q_1^2 + \ldots\ldots}{p_0^1 q_1^1 + p_0^2 q_1^2 + \ldots\ldots} \times 100 = \frac{\sum_i p_1^i q_1^i}{\sum_i p_0^i q_1^i} \times 100$$

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Revealed preference and Index Number

Price Index

- In our notation for two commodities x_1 and x_2 and instead of 100 when base year value is 1
- **Laspeyres Price Index (L^P_{01}) : weights are base year quantity** (x_0)

$$L^P_{01} = \frac{p^1_1 x^1_0 + p^2_1 x^2_0}{p^1_0 x^1_0 + p^2_0 x^2_0}$$

- **Paasche Price Index (P^P_{01}) : weights are current year quantity** (x_1)

$$P^P_{01} = \frac{p^1_1 x^1_1 + p^2_1 x^2_1}{p^1_0 x^1_1 + p^2_0 x^2_1}$$

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Welfare analysis using Laspeyres Price Index Number

Price Index

- Can we use these indexes to make welfare statement ?
- Note that comparison with 1 is not possible for the price index, since price of numerator and denominator are different.
- Let us define a new index of the relative change in total expenditure or income $m \equiv \frac{p_1^1 x_1^1 + p_1^2 x_1^2}{p_0^1 x_0^1 + p_0^2 x_0^2}$
- **Laspeyres Price Index (L_{01}^P)**

$$L_{01}^P = \frac{p_1^1 x_0^1 + p_1^2 x_0^2}{p_0^1 x_0^1 + p_0^2 x_0^2} < m \therefore \frac{p_1^1 x_0^1 + p_1^2 x_0^2}{p_0^1 x_0^1 + p_0^2 x_0^2} < \frac{p_1^1 x_1^1 + p_1^2 x_1^2}{p_0^1 x_0^1 + p_0^2 x_0^2}$$
$$\implies p_1^1 x_0^1 + p_1^2 x_0^2 < p_1^1 x_1^1 + p_1^2 x_1^2$$

- WARP imply that consumers are **better off** now.
- Ambiguous results when $L_{01}^P > m$

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Welfare analysis using Paasche Price Index Number

Price Index

- **Paasche Price Index (P_{01}^{P})**

$$
\begin{aligned}
P_{01}^{P} &= \frac{p_1^1 x_1^1 + p_1^2 x_1^2}{p_0^1 x_1^1 + p_0^2 x_1^2} > m \\
\implies p_0^1 x_0^1 + p_0^2 x_0^2 &> p_0^1 x_1^1 + p_0^2 x_1^2
\end{aligned}
$$

- WARP imply that consumers are **worse off** now. This is quite intuitive. After all, if prices rise more than income rises in the movement from base year 0 to current year 1 we would expect that would tend to make the consumer worse off.
- Ambiguous results when $P_{01}^{P} < m$.

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Classification of Index Number

Quantity Index

- Quantity Index number can also be classified in two major groups. (1) Aggregative , (2) Relative. Each of them is again classified into two groups (i) Unweighted and (ii) Weighted.
- **(1) Aggregative - (i) Unweighted :**

$$I_{01}^{uw} = \frac{q_1^1 + q_1^2 + \dots\dots}{q_0^1 + q_0^2 + \dots\dots} \times 100 = \frac{\sum_i q_1^i}{\sum_i q_0^i} \times 100$$

- **- (ii) Weighted :**

$$I_{01}^{w} = \frac{q_1^1 w_1 + q_1^2 w_2 + \dots\dots}{q_0^1 w_1 + q_0^2 w_2 + \dots\dots} \times 100 = \frac{\sum_i q_1^i w_i}{\sum_i q_0^i w_i} \times 100$$

Classification of Index Number

Quantity Index

- **Laspeyres Quantity Index (L_{01}^{Q}) : weights are base year price** (p_0)

$$L_{01}^{Q} = \frac{q_1^1 p_0^1 + q_1^2 p_0^2 + \ldots\ldots}{q_0^1 p_0^1 + q_0^2 p_0^2 + \ldots\ldots} \times 100 = \frac{\sum_i q_1^i p_0^i}{\sum_i q_0^i p_0^i} \times 100$$

- **Paasche Quantity Index (P_{01}^{Q}) : weights are current year quantity** (p_1)

$$P_{01}^{Q} = \frac{q_1^1 p_1^1 + q_1^2 p_1^2 + \ldots\ldots}{q_0^1 q_1^1 + q_0^2 p_1^2 + \ldots\ldots} \times 100 = \frac{\sum_i q_1^i p_1^i}{\sum_i q_0^i p_1^i} \times 100$$

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Welfare analysis using Laspeyres Quantity Index Number

Quantity Index

- **Laspeyres quantity Index (L_{01}^Q)**

$$L_{01}^Q = \frac{p_0^1 x_1^1 + p_0^2 x_1^2}{p_0^1 x_0^1 + p_0^2 x_0^2} < 1$$
$$\implies p_0^1 x_1^1 + p_0^2 x_1^2 < p_0^1 x_0^1 + p_0^2 x_0^2$$

- WARP imply that consumers are **worse off** now.
- Ambiguous results when $L_{01}^Q > 1$

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Welfare analysis using Paasche Quantity Index Number

Quantity Index

- **Paasche quantity Index (P_{01}^{Q})**

$$P_{01}^{Q} = \frac{p_1^1 x_1^1 + p_1^2 x_1^2}{p_1^1 x_0^1 + p_1^2 x_0^2} > 1$$
$$\implies p_1^1 x_1^1 + p_1^2 x_1^2 > p_1^1 x_0^1 + p_1^2 x_0^2$$

- WARP imply that consumers are **better off** now. Since they could have consumed the '0' consumption bundle in the '1' situation but chose not to do so.
- Ambiguous results when $P_{01}^{Q} < 1$

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Assignments

- **Assignment 3:** When two commodity baskets are purchased by the consumer at two different points of time, explain how price weighted quantity indices may be used to verify the Weak Axiom of Revealed Preference. (5 marks, CU 2020)
- **Assignment 4:** The Utility function of Debasis is $U = C_1 C_2$. His income and prices of the commodities in two periods are as follows :

Period	Income	Price of C_1	Price of C_2
1	200	20	20
2	200	20	50

Calculate the Laspeyre's index.(2 marks, CU 2020)

Uncertainty

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Choice Under Uncertainty

- Individuals or firms make choices under uncertainty. For example, when a consumer is buying a second hand car in a lemon market, lets say, 50% chance that it could be in good condition and 50% chance it could be in bad condition. And hence the quality of the car is uncertain. Therefore the resale value of the car is also uncertain.
- When a farmer is producing crops the future selling price of the crops is uncertain. If the expected aggregate production is good the expected price is low. And hence the expected profit.

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Lottery or Gambles

- Uncertain event with associated probability can be represented by a lottery or gambles.
- Let $A = \{a_1, a_2, ..., a_n\}$ denote a finite set of **outcomes**. Where each outcomes a_i occurs with an associated probability $p_i \in [0, 1]\ \forall i = 1, 2, , .., n$ and sum of these probabilities satisfies $\sum_i^n p_i = 1$. Then the set of simple gambles (on A), is given by

$$\mathcal{G}_s \equiv \left\{ (p_1 \circ a_1, ..., p_n \circ a_n) | p_i \geq 0, \sum_{i=1}^{n} p_i = 1 \right\}$$

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Simple Lottery

- Suppose that you have entered into the following bet with a friend. If the toss of a fair coin comes up heads, she pays you one rupee and you pay her one rupee if it is tails. From your point of view, this gamble will result in one of the two outcomes and hence $A = \{1, -1\}$, where 1 means win one rupee and -1 means loosing one rupee, and $p(1) = 1/2$ and $p(-1) = 1/2$ and

$$\mathcal{G}_s \equiv \left\{ \frac{1}{2} \circ 1, \frac{1}{2} \circ -1 | p_i \geq 0, \sum_{i=1}^{n} p_i = 1 \right\}$$

$$\implies \mathcal{G}_s \equiv \left(\frac{1}{2}, -\frac{1}{2} \right)$$

Compound Lottery

- A compound lottery is a lottery of various lotteries. It is a collection of lotteries. Suppose the lottery G_1 occurs with probability α_1 and the lottery G_2 occurs with probability α_2 and so on. Then the compound lottery is

$$\mathcal{G}_c \equiv \left\{ (\alpha_1 \circ G_1, ...\alpha_k \circ G_k) | \alpha_i \geq 0, \sum_{i=1}^{k} \alpha_i = 1 \right\}$$

- Let $G_1 = (1, 0, 0)$ and $G_2 = (\frac{1}{4}, \frac{3}{8}, \frac{3}{8})$. Both are independent and $\alpha_1 = \frac{1}{3}$ and $\alpha_2 = \frac{2}{3}$. Therefore the reduced form of the lottery can be written as follows: First element $= 1 \times \frac{1}{3} + \frac{1}{4} \times \frac{2}{3} = \frac{1}{2}$, Second element $= 0 \times \frac{1}{3} + \frac{3}{8} \times \frac{2}{3} = \frac{1}{4}$ and third element $= 0 \times \frac{1}{3} + \frac{3}{8} \times \frac{2}{3} = \frac{1}{4}$. So the combined or reduced form of the lottery is $\mathcal{G}_c = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$.

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Random Variable

Expected Value

- Let us consider tossing of a fair coin thrice. The set of outcomes is {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT }. Let us define a variable $X =$ **number of heads** in the trial . So X can take value $x = 0$, $x = 1$, $x = 2$, and $x = 3$ with corresponding probabilities $\frac{1}{8}, \frac{3}{8}, \frac{3}{8},$ and $\frac{1}{8}$ respectively . So X is called a random variable.
- A lottery can be thought as a random variable with associated probabilities. Like any random variable we can derive the expected value of a lottery.
- **Expected Value (EV)**: The average payoff of a lottery, where each payoff is weighted by its associated probability. So the EV, therefore, computes the average payoff with its associated probability of occuring.

Finding the Expected Value of a lottery

- **Finding the EV of a lottery** : Consider the following lottery, outcome A (₹90) occurs with probability 0.1 , outcome B (₹20) occurs with probability 0.6 and outcome C (₹60) occurs with probability 0.3. The EV of the lottery is given by the weighted average

$$\begin{aligned} EV &= (0.1 \times 90) + (0.6 \times 20) + (0.3 \times 60) \\ &= 39 \end{aligned}$$

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How Expected Value helps in decision making

- Consider the following lottery L_1, outcome A (₹100) occurs with probability $\frac{1}{3}$, outcome B (₹90) occurs with probability $\frac{2}{3}$. The EV of the lottery L_1 is given by the weighted average

$$\begin{aligned} EV(L_1) &= \frac{1}{3} \times 100 + \frac{2}{3} \times 90 \\ &= 93\frac{1}{3} \end{aligned}$$

- Lottery L_2, outcome A (₹100) occurs with probability $\frac{2}{3}$, outcome B (₹90) occurs with probability $\frac{1}{3}$. The EV of the lottery L_2 is given by the weighted average

$$\begin{aligned} EV(L_2) &= \frac{2}{3} \times 100 + \frac{1}{3} \times 90 \\ &= 96\frac{2}{3} \end{aligned}$$

- Lottery L_2 is better as it gives higher expected value compare to lottery L_1

Measuring the riskiness

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- While the EV informs about the expected payoff of a lottery, it does not provide us with a measure of how risky the lottery is. For instance, a lottery with two equally likely outcomes D (₹30) and E(₹48) also generates an EV of ₹39.

$$
\begin{aligned}
EV &= (0.5 \times 30) + (0.5 \times 48) \\
&= 39
\end{aligned}
$$

- Intutively while the lottery in previous example has a large variability of payoofs (with payoffs ranging from ₹20 to ₹90).
- One measure of riskiness of a lottery is its **variance**.

Variance of Lottery

- The variance of the risky asset

$$
\begin{aligned}
Var_{risky} &= 0.1 \times (90-39)^2 + 0.6 \times (20-39)^2 \\
&+ 0.3 \times (60-39)^2 \\
&= 609
\end{aligned}
$$

- Whereas the variance of relatively safer lottery is

$$
\begin{aligned}
Var_{safe} &= 0.5 \times (30-39)^2 + 0.5 \times (60-39)^2 \\
&= 81
\end{aligned}
$$

- Higher the variance of an asset higher the riskiness.

General Formula of EV and variance of a random variable

- Expected Value (EV) : If $x_i's$ are the values of a random variable X and $p_i's$ are the corresponding probabilities then expected value (EV) can be calculated as

$$EV\ of\ X \equiv E(X) = \sum_i p_i x_i \;\; i = 1, 2, ..., n$$

- If $x_i's$ are the values of a random variable X, $E(X)$ is the expected value of it and $p_i's$ are the corresponding probabilities then variance can be calculated as

$$\begin{aligned} var\ of\ X \equiv var(X) &= E[x_i - E(X)]^2 \\ &= \sum_i p_i (x_i - E(X))^2 \\ &= \sum_i p_i (x_i - \sum_i p_i x_i)^2 \;\; i = 1, 2, ..., n \end{aligned}$$

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St Petersburg Paradox

- Suppose you are engaged with a lottery with your friend as follows: If the coin lands head on the first flip you win ₹1. If it lands on the second flip you win ₹2 , if it lands on the third flip you win ₹4 and so on. If the entry fee of the lottery is $0 < x < \infty$, will you play the game? [1] The probabilities of the outcomes are $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$..... The expected monetary value of the St Petersburg game is

$$
\begin{aligned}
&= \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \frac{1}{8} \times 4.....\infty \\
&= \frac{1}{2} + \frac{1}{2} + \frac{1}{2}....\infty \\
&= \sum_{i}^{\infty} \left(\frac{1}{2}\right)^{n} 2^{n-1} \\
&= \infty
\end{aligned}
$$

[1] A very brief history is nicely explained here https://plato.stanford.edu/entries/paradox-stpetersburg/

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St Petersburg Paradox : Reconciliation

- suppose you are asked to pay ₹1000 will you play the game ?
- Then what should be the optimum entry fee?
- One way to reconcile this paradox is to propose an upper boundary of the outcome value. Suppose that the upper boundary of an outcome's value is 2^m . If so, that outcome will be obtained if the coin lands heads on the m^{th} flip. This means that the expected value of all the infinitely many possible outcomes in which the coin is flipped more than m times will be finite: It is 2^m times the probability that this happens, so it cannot exceed 2^m.

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St Petersburg Paradox : Reconciliation

- Another way is to suggest that individuals are **risk-averse**. This is the approach taken by two eighteenth century mathematicians Daniel Bernoulli and Cramér.
- Cramér was aware that it would be controversial to claim that there exists an upper boundary beyond which additional riches do not matter at all. However, he pointed out that his solution works even if the value of money is strictly increasing but the relative increase gets smaller and smaller (Concave function) (21 May 1728): *If one wishes to suppose that the moral value of goods was as the square root of the mathematical quantities . . . my moral expectation will be*

$$= \frac{1}{2} \times \sqrt{1} + \frac{1}{4} \times \sqrt{2} + \frac{1}{8} \times \sqrt{4}.....\infty$$

St Petersburg Paradox : Reconciliation

- Cramér correctly calculated the expected utility ("moral value") of the St. Petersburg game to be about 2.9 units for an agent whose utility of money is given by the root function.
- This expected utility proposed by Cramér was later formed into formal theory in the hand of Von Neumann and Morgenstein (1944).

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Expected Utility

- **Expected Utility** The average utility of a lottery, weighting each utility with the associated probability of that outcome.
- Example : Consider an individual with utility function $U(W) = \sqrt{W}$, where $W \geq 0$ denotes the income or wealth that the individual receives in each outcome. Let us calcuate the EU of previous risky asset example

$$\begin{aligned} EU_{risky} &= \left(0.1 \times \sqrt{90}\right) + \left(0.6 \times \sqrt{20}\right) + \left(0.3 \times \sqrt{60}\right) \\ &= 5.96 \end{aligned}$$

- While that of the second (less risky lottery) is

$$\begin{aligned} EU_{safe} &= \left(0.5 \times \sqrt{30}\right) + \left(0.5 \times \sqrt{48}\right) \\ &= 6.20 \end{aligned}$$

- This indicates that the individual obtains a higher expected utility from the second lottery. While both lotteries generates same expected value.

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Expected Utility

State Preference Approach

- Suppose there are only two possible states , state 1 and state 2. The probability of state 1 is π_1 and probability of state 2 is π_2 such that $\pi_1 + \pi_2 = 1$. Let W_1 and W_2 denote the wealth contingent upon state 1 and state 2, respectively. Under some important axioms we can find a utility function $u(.)$ such that

$$U(W_1, W_2; \pi_1, \pi_2) = \pi_1 u(W_1) + \pi_2 u(W_2) \qquad (7)$$

  The function $u(.)$ is called *Von Neumann- Morgenstein utility function*. Note that preferences are now expressed as the expected value of a utility function.
- This representation of preferences is simple because utility is **additively separable** in W_1 and W_2 and is **linear** in π_1 and π_2.

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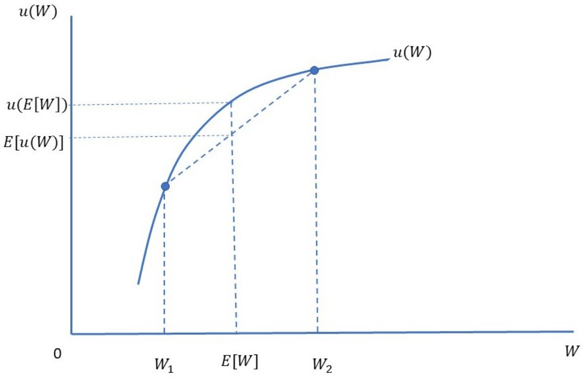
Risk attitude : Risk Averse

- A person is said to be “risk averse” if she prefers to receive the EV of the lottery with certainty (called certainty equivalent), where she obtains $u(EV)$, rather than having to face the risk of playing the lottery, which yields EU.



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Risk aversion

- Here the EV is $E[W] = \pi_1 W_1 + \pi_2 W_2$, $u(E[W]) = u(\pi_1 W_1 + \pi_2 W_2)$, and $E[u(W)] = \pi_1 u(W_1) + \pi_2 u(W_2)$
- For any concave function $f(x)$ if x_1 and x_2 are two arbitrary point and for any $(0 \leq \theta \leq 1)$ we know

$$f(\theta x_1 + (1-\theta)x_2) \geq \theta f(x_1) + (1-\theta) f(x_2)$$

- **Jensen's Inequality** We can rewrite above equation as

$$f(\theta_1 x_1 + \theta_2 x_2) \geq \theta_1 f(x_1) + \theta_2 f(x_2)$$

  Where $(\theta_1 + \theta_2 = 1)$.
- So for any concave function from Jensen's Inequality we can prove that $u(E[W]) = u(\pi_1 W_1 + \pi_2 W_2) \geq E[u(W)] = \pi_1 u(W_1) + \pi_2 u(W_2)$ which is the condition of risk aversion.

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Risk attitude

- So in summary we can say if a person is **risk averse** her utility function is **concave** or vice-versa (for example $u(W) = \sqrt{W}, u(W) = log_e(W)$ etc.
  that is where $u''(W) < 0$)
- If the person is **risk neutral** her utility function is both **convex and concave** that is **linear.**
  (that is where $u''(W) = 0$)
- And finally, if the person is **risk loving** her utility function is **convex**. (for example $u(W) = W^2 \ \forall \ W \geq 0$
  that is where $u''(W) > 0$)

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Risk Premium (RP)

- The amount of money that we need to substract from the EV in order to make the decision maker indifferent between playing the lottery and accepting the EV from the lottery. That is the RP solves

$$u(EV - RP) = EU$$

- consider the safe lottery example, recalling that $EV = 39$ and $EU = 6.20$ whose utility function is $u(W) = \sqrt{W}$, the RP solves

$$\begin{aligned} u(39 - RP) &= 6.20 \\ \implies \sqrt{39 - RP} &= 6.20 \\ \implies 39 - RP &= (6.20)^2 \\ \implies RP &= 0.56 \end{aligned}$$

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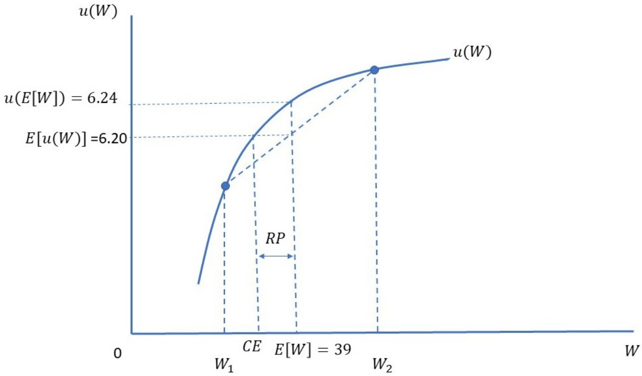
Risk Premium (RP)



Certainty Equivalent:(CE)

- The amount of money that, if given to the individual with certainty makes her indifferent between receiving such a certain amount and playing the lottery. That is

$$CE = \quad EV - RP$$

- In previous example $CE = EV - RP = 39 - 0.56 = 38.44$. That is if we offer ₹38.44 to the risk averse individual whose utility function is $u(W) = \sqrt{W}$, she would be indifferent between receiving this amount and playing the lottery.

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Measures of Risk Aversion

Absolute Risk Aversion

- As we have seen that concavity of the utility function (which means the convexity of the indifference curve) imply risk aversion. A natural measure of the degree of risk aversion is therefore the degree of concavity (the curvature) of the utility function or the degree of convexity of the indifference curve.
- The degree of concavity of the utility function can be measured by the ratio $-u''(W)/u'(W)$. We call this quantity the **Arrow-Prat measure of absolute risk aversion** (R_a)**.**

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Measures of Risk Aversion

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Absolute Risk Aversion : Derivation

- Suppose an individual has initial wealth W. A risk averse individual will not be willing to take a fair gamble. The risk premium $P_x(W)$ is defined as the amount a person is willing to pay to avoid a fair gamble x (with mean 0 and variance σ_x^2). mathematically we can write

$$u(W - P_x(W)) \equiv E[u(W + x)]$$

Taking a first-order Taylor series approximation on the left and second order approximation on the right, we obtain

$$u(W) - P_x(W)u'(w) \quad \approx E\left[u(W) + xu'(W) + \tfrac{1}{2}x^2u''(W)\right]$$

$$\approx u(W) + \frac{1}{2}\sigma_x^2 u''(W)$$

Measures of Risk Aversion

And therefore

$$P_x(W) \approx \frac{1}{2}\sigma_x^2 \frac{-u''(W)}{u'(w)} \tag{8}$$

Thus, the higher the coefficient of absolute risk aversion, the higher the risk premium the individual is willing to pay.

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Measures of Risk Aversion

Relative risk aversion

$$u(W - W\hat{P}_x(W)) \equiv E[u(W + Wx)]$$

Taking Taylor Series Approximation on both sides

$$u(W) - W\hat{P}_x(W)u'(w) \approx E\Big[u(W) + Wxu'(W) + \frac{1}{2}W^2x^2u''(W)$$

and therefore

$$P_x(W) \approx \frac{1}{2}\sigma_x^2 \frac{-Wu''(W)}{u'(w)}$$

Again, the relative risk premium is higher as the coefficient of **relative risk aversion** higher.

Measures of Risk Aversion : example

Relative risk aversion

- An example of utility function having constant relative risk aversion is

$$u(W) = W^{\alpha}$$

A concave utility function means $0 < \alpha < 1$. The coefficient of risk aversion can be calculated as

$$R_r = \frac{-W\alpha(\alpha - 1)W^{\alpha-2}}{\alpha W^{\alpha-1}} = 1 - \alpha$$

- Where R_r, the coefficient of relative risk aversion, is constant in wealth , W.
- **Assignment 1**: Prove that the utility function

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

(a) has CRRA property and the coefficient is σ, (b) she is prudent . (A prudence means $\frac{dR_a}{dW} < 0$) the coefficient of absolute risk aversion (R_a) is decreasing in W)

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Measures of Risk Aversion : Assignments

- **Assignment 2**: Comment on the attitude toward risk for the following utility functions
  (i) $u = a - be^{-rW}; a, b, r > 0$
  (ii) $u = a + b\ lnW; a, b > 0$
  (iii) $u = a + b\frac{W^r}{r}; 0 < r < 1; a, b > 0$
  (iv) $u = W(1 - e^{-W})$
- **Assignment 3** : A person has an expected utility function of the form $u(W) = \sqrt{W}$. He initially has wealth of ₹4. He has a lottery ticket that will be worth ₹12 with probability half and will be worth zero with probability half. What is his expected utility ? What is the lowest price p at which he would part with the ticket ? (hint: $\sqrt{4 + p} = 3$)

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Measures of Risk Aversion : Assignments

- **Assignment 4:** (a) Suppose Natasha's utility function is given by $u(I) = \sqrt{10I}$, where I represents annual income in thousand of rupees.
  (i) Is Natasha Risk loving, risk neutral or risk averse ?
  (ii) Suppose that Natasha is currently earning an income of ₹40,000 (I = 40) and can earn that income next year with certainty. She is offered a chance to take a new job that offers a 0·6 probability of earning ₹44,000 and 0·4 probability of earning ₹33,000. Should she take the new job?— Why?
  (b) (i) What do you mean by risk premium?
  (ii) Irma is risk-averse. She gets an expected utility of 105 utils from a lottery with expected income of ₹4,000. However, she gets an utility of 105 utils from a certain wealth of ₹2,600 only. Calculate her risk premium.

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